STAT GU4207 / GR5207 HOMEWORK #1

SPRING 2025

Assigned: Wed, Jan 22
Due: Fri, Jan 31

1. Suppose that X is an exponential random variable with rate $\alpha > 0$, that Y is an exponential random variable with rate $\beta > 0$, and that X and Y are independent. Show
$$\mathbb{P}(X < Y) = \frac{\alpha}{\alpha + \beta}$$

2. Suppose that $X_0, X_1, \ldots$ are i.i.d. continuous random variables, and define the random variable $N := \min\{n \geq 1 : X_n > X_0\}$, which is the index of the first random variable to exceed X_0. Compute $\mathbb{E}[N]$.

3. Suppose that N has a Poisson distribution with rate 1, and that the conditional distribution of X given $\{N = n\}$ is uniform over $\{0, 1, \ldots, n+1\}$. Find the marginal distribution of X.

4. Suppose that Λ has an exponential distribution with rate $\theta > 0$, and that the conditional distribution of Y given $\{\Lambda = \lambda\}$ is Poisson with rate λ. Find the conditional distribution of Λ given $\{Y = k\}$.

5. Suppose X and Y are independent random variables, and that they both have geometric distribution with success probability $0 \leq p \leq 1$,
$$\mathbb{P}(X = k) = \mathbb{P}(Y = k) = (1 - p)p^k$$
for all $k \in \mathbb{N}$. Set $U = \min\{X, Y\}, V = \max\{X, Y\}$, and $W = V - U$. Find the marginal distribution of U, and show that U and W are independent.