STAT GU4207 / GR5207 HOMEWORK #5

SPRING 2025

Assigned: Wed, Feb 19
Due: never

1. Suppose $Y_1, Y_2, \ldots$ are successive flips of a fair coin, where H denotes heads and T denotes tails. What is the expected number of required flips until the most recent three outcomes show the pattern HTH? (Recall that in lecture we did a similar calculation for the pattern HHT. Why are these answers different?)

2. Let $\{X_n\}_{n\geq 0}$ be a simple symmetric random walk and let $T_M := \min\{n \geq 0 : X_n \in \{0, M\}\}$ denote the first exit time from $\{1, 2, \ldots, M-1\}$. Compute the value
$$\mathbb{E}[T_M \,|\, X_0 = M/2],$$
where M is assumed to be even.

3. For a fixed positive integer M, let $\{X_n\}_{n\geq 0}$ denote the Ehrenfest gas model on M particles. Let $T_0 := \min\{n \geq 1 : X_0 = 0\}$ be the first time returning to the state where all particles are in the right chamber. Compute the value
$$\mathbb{E}[T_0 \,|\, X_0 = 0]$$

4. Let $\{Z_n\}_{n\geq 0}$ denote a branching process with offspring distribution p. Compute
$$\mathbb{E}\left[\sum_{n=0}^{\infty} Z_n\right],$$
which is expected value of the total number of individuals over all time.

5. Consider a branching process where even generations follow an offspring distribution p and odd generations follow an offspring distribution q. Find the extinction probability of this branching process, in terms of the probability generating functions of p and q.